"ITMP, MSU, 16.12.20" - - ' '

Loops in Holography

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QFT on $\mathbb{R}^{3,1}$ $\rightsquigarrow$ S-matrix $\leftarrow$ {Spins, (ren.) masses, amplitudes}

rep. of Poincaré

$\updownarrow$

QFT on $AdS^{3,1}$ $\rightsquigarrow$ correlation functions $\leftarrow$ {Spins 〃, (ren.) dimensions, operator product coeff.}

CFT

rep. of conformal group, $SO(3,2)$ $= ISO(AdS_4)$

conjectures

$\boxed{\text{QFT}}$ on $\text{AdS}^{(4)}_{\;3,1}$ $\rightsquigarrow$ $\boxed{\text{CFT}}$

strings

higher spin (Vassiliev)

$\mathcal{N}=4$ SUSY Yang Mills (Maldacena '98)

...

O(N) model (Klebanov, Polyakov '02)

∴ for a partial construction of HS see: Baekaert, Erdmenger, Ponomarev, Sleight '15

This talk:

$$\boxed{\text{given a QFT on } \begin{Bmatrix} \text{EAdS}_4 \\ \text{dS}^{3,1} \end{Bmatrix} \text{ construct its CFT.}}$$

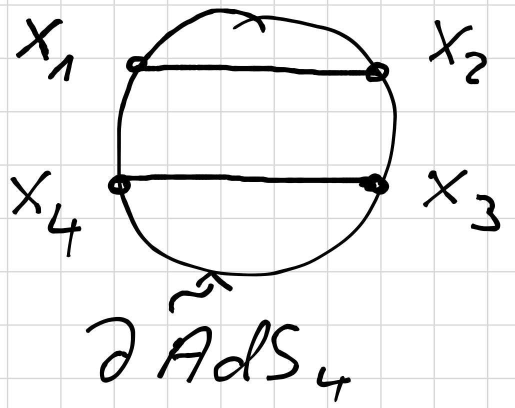
Questions:

(1) systematics : loop expansion in (A)dS

(2) what is the use: generating function of hopefully (useful) CFT's

(3) Physical applications: $\begin{cases} \text{relation btw. CFT's} \\ \text{de Sitter ?} \end{cases}$

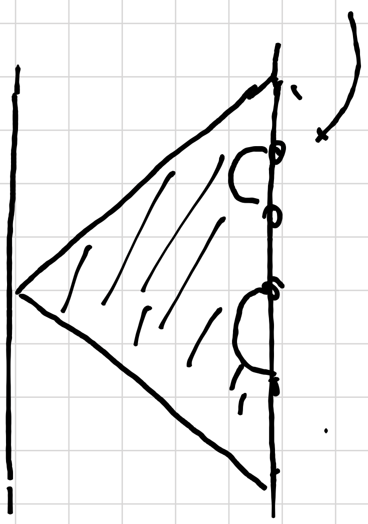
(1) Consider an interacting scalar field φ on AdS_4, here $V(\varphi) = m^2\varphi^2 + \lambda\varphi^4$

$O(\lambda^0)$: 

$+ \text{perm}\{x_i\}$: $G(x_1 \ldots x_4) = \underbrace{\langle O_\Delta(x_1) \cdots O_\Delta(x_4)\rangle}_{\parallel}$

(conformal boundary) $= \sum \langle O_\Delta(x_1)\, O_\Delta(x_2)\rangle \langle O_\Delta(x_3)\, O_\Delta(x_4)\rangle$

$+ \text{perm}\{x_i\}$



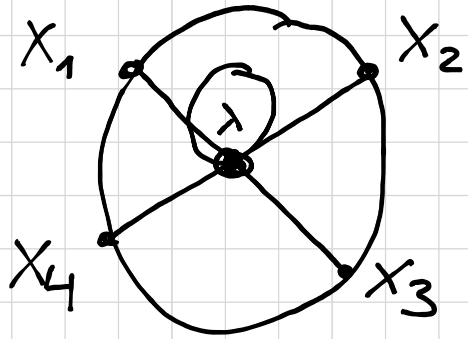
$$\frac{1}{|x_1 - x_2|^{2\Delta}}$$

$\boxed{\text{Generalised free field (Greenberg '61)}}$

Systematics: perturbative exp'n in λ $\rightsquigarrow$ non-Gaußian def'n of the gen. free field

(Heemskerk, Penedones, Polchinski, Sully '09)

$O(\lambda)$:



: $G^{(1)}(x_1 \cdots, x_4)$ manifests itself in CFT through:

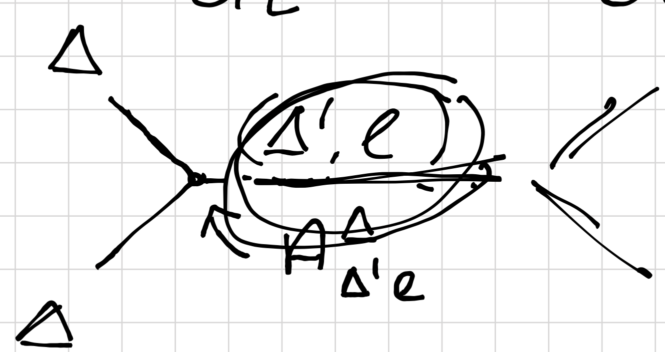
(Mueck, Viswanathan '98)

$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}, \quad v = \cdots \quad \Big\}\ \text{cross ratios}$$

$$\underbrace{\boxed{O_\Delta(x_1)\, O_\Delta(x_2)}}_{\text{OPE}}\ \underbrace{\boxed{O_\Delta(x_3)\, O_\Delta(x_4)}}_{\text{OPE}} \sim \frac{1}{|x_{12}|^{2\Delta}} \frac{1}{|x_{34}|^{2\Delta}} \boxed{G(u,v)}$$

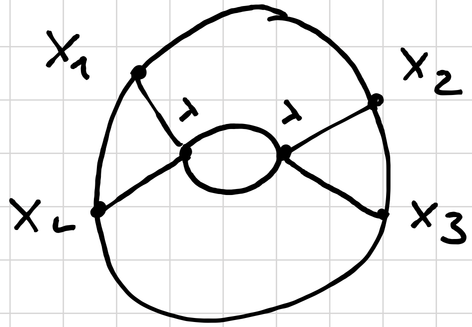
$$G(u,v) = 1 + \sum_{\Delta', l} u^{\frac{1}{2}(\Delta' - l)}\, g_{\Delta' l}(u,v)$$

conf. blocks (Dolan, Osborne)



$$\boxed{O_{\Delta' l} = O_\Delta\, \Box^m \partial_i^l\, O_\Delta, \quad \Delta' = \Delta'(\lambda), \quad A^{\Delta}_{\Delta' l}(\lambda)}$$

$O(\lambda^2)$:



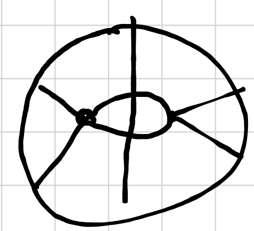
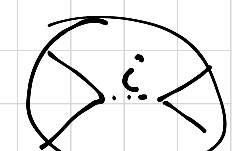
is hard!

- position space, e.g. 

$\sim \int \underline{H^{(2)}(\underline{x}-\underline{y})}\, \underline{H^{(2)}(\underline{y}-\underline{z})}\; \delta m^2 \, dy$
- use Schwinger parameter
- use Mathematica.

(Bertan, I.S., Skvortsov '18)

Alternative approach:
use unitarity cuts

 $= \sum_i$ 

(Fitzpatrick, Kaplan '12, ...)

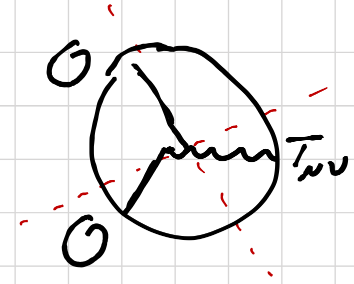
Results:

$$\boxed{\Delta_{0,l} = 4 + l + \gamma\delta_{l,0} + \gamma^2 \begin{cases} \frac{5}{3} & \text{for } l = 0, \\ -\frac{6}{(l+3)(l+2)(l+1)l} & \text{for } l > 0, \end{cases} \qquad \gamma = -\lambda_R/16\pi^2}$$

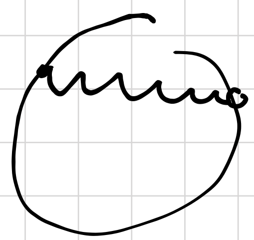
1-loop contribution

Rem: • w/ P. Vanhove we learned how to simplify this enormously . . .

• no dynamical gravity!



⟷ CFT w/ stress tensor!



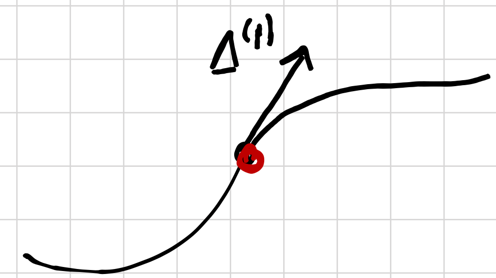
$\langle \cancel{T_{\mu\nu}}\, \cancel{T_{\mu\nu}} \rangle$

g. $\boxed{S_{CFT} = \iint \frac{\phi(x)\,\phi(y)}{|x-y|^{*}} + \int t\,\phi^2 + \int g\,\phi^4}$

• renormalisability : ⟷ predictivity :

$\Delta^{(2)}_{i_n}(\Delta^{(1)}_{(n)})$

$\boxed{\Delta^{(2)} = \underline{\Delta}^{(2)}(\underline{\Delta}^{(1)}, \cancel{\Lambda})}$



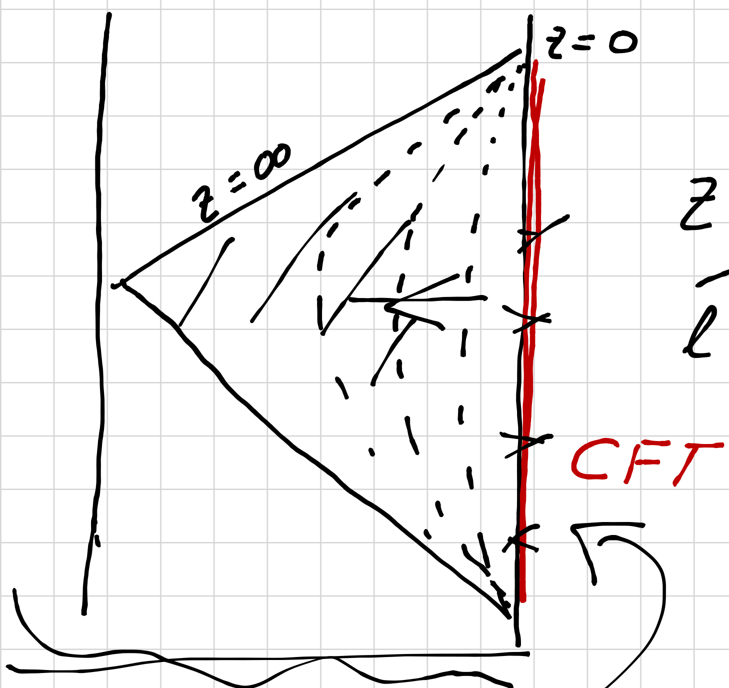
(1) systematics ✓✓✓

(2) useful for generating CFT's (are they useful?)

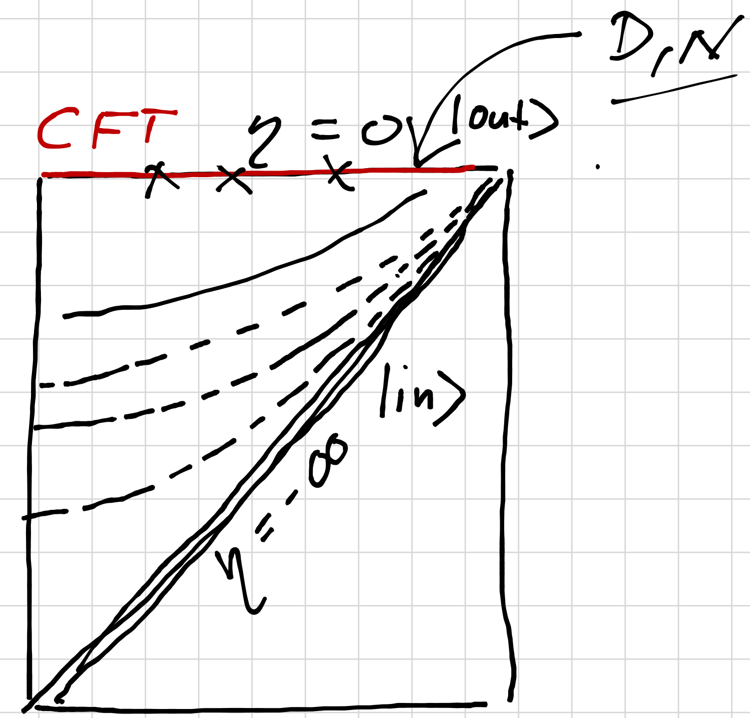
• *Alternative appr.*: reverse engineer result with conformal bootstrap (e.g. Aharony, Alday, Aprile, Bissi, Drummond, Heslop, Perlmutter, ...)

(3) de Sitter:

$EAdS_\ell$



$$z = -i\eta \quad \longrightarrow \quad \ell = i\ell_{dS}$$



Green function:

$$G^{EAdS}_{\underset{w}{D},\underset{u}{N}}(x,x') \longrightarrow G^{dS}_{N,D}(\underline{x},\eta,\underline{x}',\eta')$$

$$= \langle \underset{D/N}{\underline{\text{out}}} | \varphi(\underline{x},\eta)\, \varphi(\underline{x}',\eta') | \underset{BD}{\underline{\text{in}}} \rangle$$

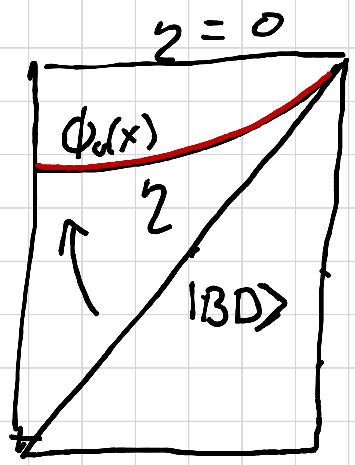
5pNdFS

EAdS implies $\begin{cases} |in\rangle = \underline{|BD\rangle} \text{ (Bunch Davies)} \\ \qquad\qquad \text{no quanta at } \eta = -\infty \\ |out\rangle = \begin{cases} |D\rangle : \underline{\lim_{\eta \to 0} \frac{1}{\eta} G(\eta, \dots) = 0} \\ |N\rangle : \lim_{\eta \to 0} \partial_\eta \frac{1}{\eta} G(\eta, \dots) = 0 \end{cases} \end{cases}$

Thus: $\langle \phi(\underline{x}) | e^{-iHt} | BD \rangle = \Psi_{BD}(\phi)$

$$Z_{EAdS}[-i\eta, \phi_0(\underline{x})]\Big|_{il_{dS}} =: \Psi[\eta, \phi_0(\underline{x})] = \text{wave function of the Bunch-Davies vacuum}$$

CFT:



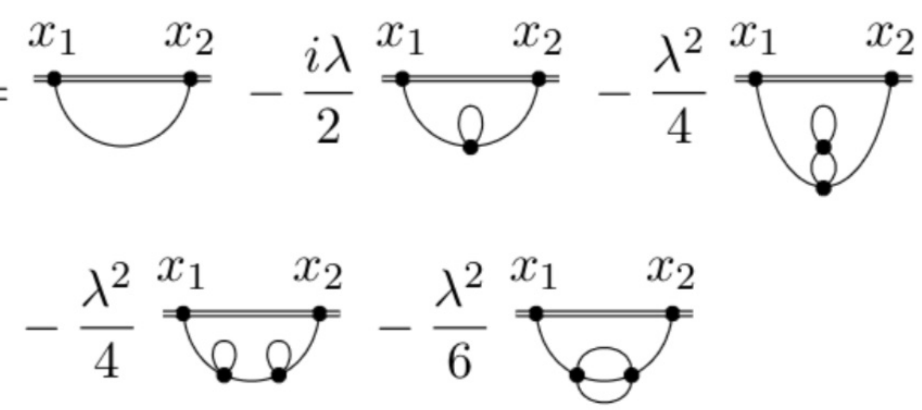
$$\langle \mathcal{O}(\underline{x}_1)\mathcal{O}(\underline{x}_2)\dots\mathcal{O}(\underline{x}_n)\rangle = \lim_{\eta \to 0} \frac{\delta^n}{\delta\phi_0(\underline{x}_1)\delta\phi_0(\underline{x}_2)\dots\delta\phi_0(\underline{x}_n)} \Psi[\eta, \phi_0(\underline{x})]$$

Evaluation: $\varphi = \phi_0 + \chi$:

$$\Psi[\phi(\underline{x}, \eta)]|_{\eta=\epsilon} = \mathrm{e}^{i\Gamma[\phi_0]} = \mathrm{e}^{iS_{on-shell}[\varphi]} \int \mathscr{D}\chi \mathrm{e}^{iS_0[\chi] + iS_{int}[\varphi, \chi]}$$

2pt function @ 1 loop:

$$\langle \mathscr{O}(\underline{x}_1)\mathscr{O}(\underline{x}_2)\rangle = \frac{\delta^2\Psi[\phi_0]}{\delta\phi_0(\underline{x}_1)\delta\phi_0(\underline{x}_2)} =$$

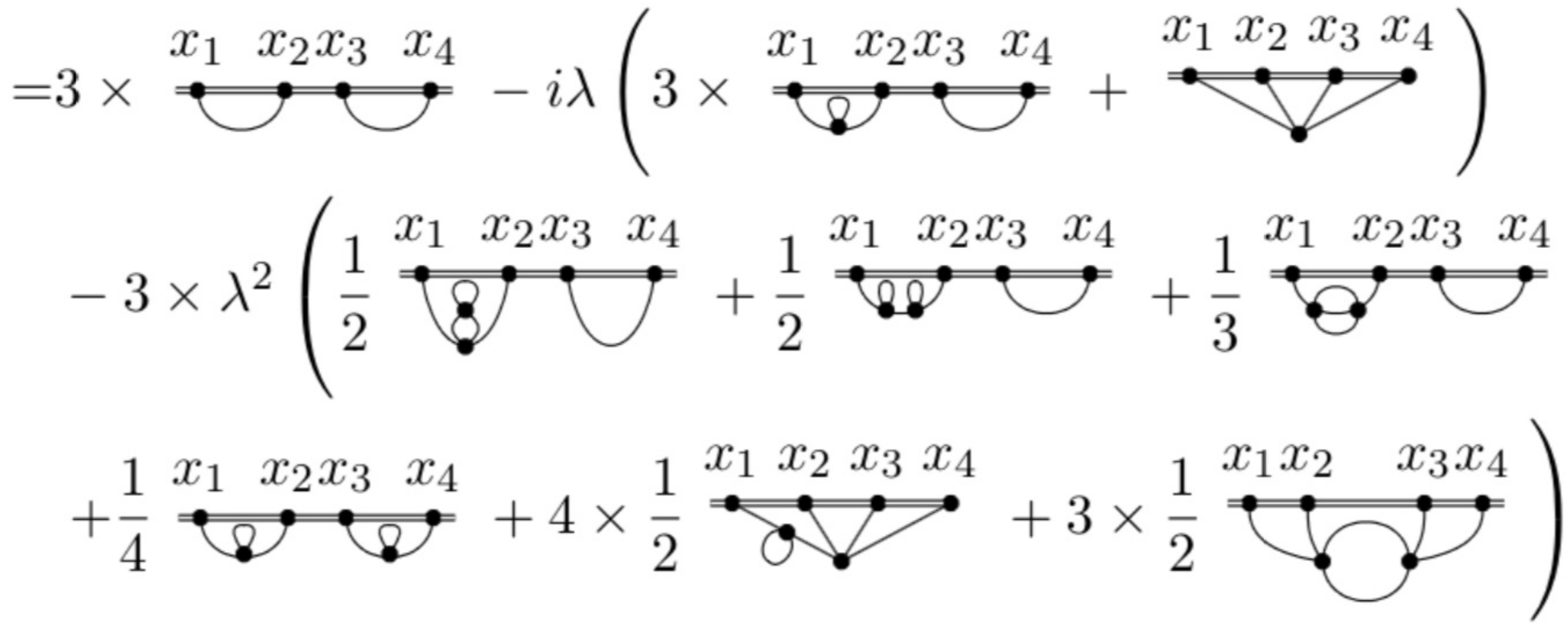


$\sim$

renormalised mass.

4-pt function:

$$\langle \mathscr{O}(\underline{x}_1)\mathscr{O}(\underline{x}_2)\mathscr{O}(\underline{x}_3)\mathscr{O}(\underline{x}_4)\rangle = \frac{\delta^4\Psi[\phi_0]}{\delta\phi_0(\underline{x}_1)\delta\phi_0(\underline{x}_2)\delta\phi_0(\underline{x}_3)\delta\phi_0(\underline{x}_4)}$$



from analytic continuation of AdS results.

Rem: • Strominger '01 : dS/CFT

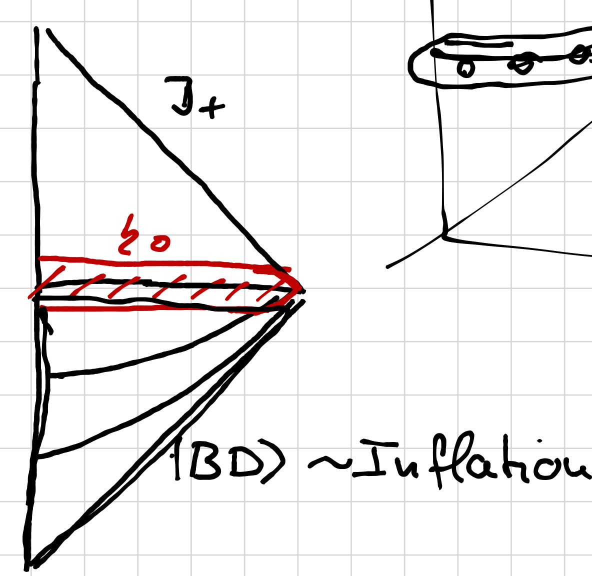
• " + Anninos + Hartman '11 $\quad AdS/O(N) \rightsquigarrow dS/SP(N)$

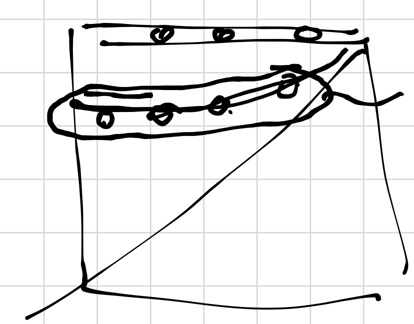
$\therefore$ note that $\begin{array}{|c|} AdS \\ \phi^4 \end{array} \longrightarrow \begin{array}{|c|} dS \\ -\phi^4 \end{array}$!

• Loops in dS cf. also Akhmedov
Senatore ~~IR~~ effect
...

Is it useful?

(I) relation to density perturbations:



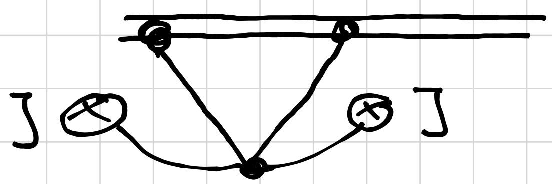


$$\langle \phi(\underline{x}_1) \cdots \phi(\underline{x}_n) \rangle =$$

$$\int [D\phi_0]\, \underline{\Psi^\dagger[z_0, \phi_0(\underline{x})]}\; \phi_0(\underline{x}_1) \cdots \phi_0(\underline{x}_n)\; \underbrace{\Psi[z_0, \phi_0(\underline{x})]}_{\text{beyond reach for now.}}$$

(II) 4-pt fn necessary? ~ non Gaußianities
(very small!)

But:



~ deviations from dS!

(e.g. Arkani-Hamed, Bauman, Lee, Pimentel '18)

(III) Why loops?

~ you never know ☺